KERDOCK-LIKE BENT FUNCTIONS

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Abstract. We introduce bent functions similar to bent functions whose binary representative vectors are members of the famous Kerdock code.

1. Introduction

1.1. **Elementary definitions.**

$\mathbb{F}_2$ is the finite field of order 2.
A m-boolean function is a map from $\mathbb{F}_2^m$ into $\mathbb{F}_2$.
Weight: $w(F) = \sharp\{v \in \mathbb{F}_2^m \mid F(v) = 1\}$.
Derivative: $e \in \mathbb{F}_2^m$ $(D_eF)(X) = F(X) + F(X + e)$.
Fourier coefficients:
$\hat{F}(v) = \sum_{X\in\mathbb{F}_2^m}(-1)^{F(X)+<v,X>}$ where $<,>$ inner product of $\mathbb{F}_2^m$.
The set $\{\hat{F}(v) \mid v \in \mathbb{F}_{2^m}\}$ is independant of the choice of $<,>$.

Definitions:
F is bent if: $\forall\, v \in \mathbb{F}_2^m : \hat{F}(v)$ is in $\{-2^{m/2}, 2^{m/2}\}$.
Exist only when m is even.

F is near-benf if: $\forall\, v \in \mathbb{F}_{2^m} : \hat{F}(v)$ is in $\{-2^{(m+1)/2}, 0, 2^{(m+1)/2}\}$.
Exist only when m is odd.

Bent functions were introduced by Rothaus in [6] .They are interesting for Coding Theory, Cryptology and Sequences and were the topic of a lot of works. See for instance [2], [5] Chap. 14, [7], [1].
For further use we need the following Proposition.

Proposition 1. *The distribution of the Fourier coefficients of a* $(2t-1)$*-near bent function* f *is:*

$\hat{f}(v) = 2^t$ *number of* v*:* $2^{2t-3} + (-1)^{f(0)}2^{t-2}$
$\hat{f}(v) = 0$ *number of* v*:* 2^{2t-2}
$\hat{f}(v) = -2^t$ *number of* v*:* $2^{2t-3} - (-1)^{f(0)}2^{t-2}$.

Proof. See Proposition 4 in [1]). □

1.2. **Special representations of boolean functions.**

1) Using finite fields:
$\mathbb{F}_2^m$ identified with the field $\mathbb{F}_{2^m}$.

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In this case the inner product $<,>$ of $\mathbb{F}_{2^m}$ is defined by:
$< a, x >= tr(ax)$ where tr is the trace of $\mathbb{F}_{2^m}$ over $\mathbb{F}_2$.

2) Representative vector (truth table)
Indexing $\mathbb{F}_{2^m}$ with any order $e_0, e_1, ...e_{2^m-1}$ the representative vector of a m-boolean function F is the binary vector $(F(e_i)_{i=0}^{2m-1})$.
This vector depends on the choice of the order of $\mathbb{F}_{2^m}$.

3) A two-variable representation.
This is the representation chosen by Kerdock to introduce his famous code.
We identify $\mathbb{F}_{2^{2t}}$ with the product:

$$\mathbb{F}_{2^{2t-1}} \times \mathbb{F}_2 = \{X = (u, \nu) \mid u \in \mathbb{F}_{2^{2t-1}}, \nu \in \mathbb{F}_2\}.$$

If F is a $(2t)$-boolean function then define two $(2t-1)$-boolean functions f_0, f_1, such that $f_0(u) = F(u, 0)$ and $f_1(u) = F(u, 1)$.

The two-variable representation (TVR) of F is defined by the 2-variable polynomial: $\phi_F(x, y) = (y + 1)f_0(x) + yf_1(x)$

This is a representation of F in the following sense. Since:
$\phi_F(u, 0) = f_0(u) = F(u, 0)$, $\phi_F(u, 1) = f_1(u) = F(u, 1)$.

then if $X = (u, \nu)$, with $u = 0$ or $u = 1$: $F(X) = \phi_F(u, \nu)$.

Notation: $F = [f_0, f_1]$

Let α be a primitive root of $\mathbb{F}_{_{2^{k-1}}}$. As order of $\mathbb{F}_{2^{2t}}$ we choose:

$$\mathbb{F}_{2^{2t}} : \quad (0, 0), (\alpha^0, 0) \dots (\alpha^i, 0) \dots (\alpha^{^{n/2-2}}, 0) \mid (0, 1), (\alpha^0, 1) \dots (\alpha^i, 1) \dots (\alpha^{^{n/2-2}}, 1)$$
The representative vector of $F = [f_0, f_1]$ is:

$$(f_0(0) \dots\dots f_0(\alpha^i) \dots\dots f_0(\alpha^{^{n/2-2}}) \quad f_1(0) \dots\dots f_1(\alpha^i) \dots\dots f_1(\alpha^{^{n/2-2}}))$$

1.3. **From Near-bent to Bent.**

We now characterize the f_0, f_1 when $F = [f_0, f_1]$ is bent.

Proposition 2. *(well known)*
A $(2t)$-boolean function $F = [f_0, f_1]$ is a bent if and only if:

(a) f_0 and f_1 are near-bent.

(b) $\forall u \in \mathbb{F}_{2^{2t-1}} \mid \hat{f}_0(u) \mid + \mid \hat{f}_1(u) \mid = 2^t$

Proof. A proof is given in [9],Proposition 14. □

Remark: (b) means that one of $\mid \hat{f}_0(a) \mid$ and $\mid \hat{f}_1(a) \mid$ is equal to 2^t and the other one is equal to 0.

1.4. **The Kerdock code.** :
Notation:

$Q(x) = \sum_{j=1}^{t-1} tr\big(x^{2^j+1}\big)$.

If $e \in \mathbb{F}_{2^m}$: $t_e(x) = tr(ex)$, $Q_e(x) = Q(ex)$.

Definition 3.
The Kerdock code of length 2^{2t} is the set of the representative vectors of the $2t$-boolean functions

$$F = [Q_u, Q_u + t_u] + \textit{affine-linear form}$$

with $u \in \mathbb{F}_{2^{2t-1}}$.

Example: The representative vector of $F = [Q_u, Q_u + t_u]$ is:

$m = (f_0(0)\ldots\ldots f_0(\alpha^i)\ldots\ldots f_0(\alpha^{n/2-2}) \quad f_1(0)\ldots\ldots f_1(\alpha^i)\ldots\ldots f_1(\alpha^{n/2-2}))$
with $f_0(x) = Q_u(x)$ and $f_1(x) = (Q_u + t_u)(x)$

Theorem 4. *(Kerdock)*
With the above notations:

If $u \neq 0$ then:
$F = [Q_u, Q_u + t_u]$ + affine-linear form is a Bent Function.

Proof. See [3] or [5] chapter 15. □

Remark:

From the elementary properties of bent functions, $F = [Q_u, Q_u + t_u] +$ affine-linear form is bent if and only if $[Q_u, Q_u + t_u]$ is bent. Hence we restrict our research to $[Q_u, Q_u + t_u]$.

Definition: For the sequel of the paper $[Q_u, Q_u+t_u]$ is called a Kerdock bent function.

The Kerdock code K_{2t} is a binary non-linear code with several interesting properties. For instance:
1) $[Q_u, Q_u + t_u]$ is a bent functions

2) $[Q_u, Q_u + t_u]$ $+[Q_v, Q_v + t_v]$ is a bent function.

A problem:
The question of this paper is to replace t_u in $[Q_u, Q_u + t_u]$ by another $2t-1$-boolean function, for example t_r, to get another bent function.

1.5. **Main tools.**

Definition 5.
If f is a $(2t-1)$-near-bent function then $\hat{I}_f$ is the indicator of the set $\{x \in \mathbb{F}_{2^{2t-1}} \mid \hat{f}(x) = 0\}$ where $\hat{f}$ is the Fourier transform of f.
(In other words, $\hat{I}_f(x) = 1$ if and only if $\hat{f}(x) = 0$).

The two Theorems below are the main tools of the present work.

Theorem 6. *(McGuire and Leander)*
Let f be a $(2t-1)$-near-bent function and let v be in $\mathbb{F}_{2^{2t-1}}$.
$D_v(\hat{I}_f) = 1$ if and only if $[f, f + t_v]$ is a bent-function.

Proof. See [4], Theorem 3. □

Theorem 7. *(W)*
Let f be a $(2t-1)$-near-bent function.
Let ω be in $\mathbb{F}_{2^{2t-1}}$ and let ϵ be in $\mathbb{F}_2$.
If $D_\omega f = \epsilon$ then $\hat{I}_f = t_\omega + \epsilon$.

Remark: According to the definition of $\hat{I}_f$ this lemma means that if $D_\omega f = \epsilon$ then $\hat{f}(x) = 0$ if and only if $t_\omega(x) = 1 + \epsilon$.

Proof. $\hat{f}(u) = \sum_{x \in \mathbb{F}_{2^{2t-1}}} (-1)^{f(x)+tr(ux)} = 2^{2t-1} - 2w(f + tr(ux))$.
$\hat{f}(u) = 0$ if and only if $w(f + t_u) = 2^{2t-2}$.

$D_\omega f = \epsilon$ means that $f(x + \omega) = f(x) + \epsilon$.
The transform $\tau : x \to x + \omega$ is a permutation of $\mathbb{F}_{2^{2t-1}}$ and then preserves the weight of every $(2t-1)$-Boolean function. Thus:

$$\sharp\{x \mid f(x) + tr(ux) = 1\} = \sharp\{x \mid f(x + \omega) + tr(u(x + \omega)) = 1\}.$$

$$(E)\ \sharp\{x \mid f(x) + tr(ux) = 1\} = \sharp\{x \mid f(x) + \epsilon + tr(ux) + tr(u\omega) = 1\}.$$

Now assume $tr(u\omega) + \epsilon = 1$. The right hand member of (E) is:
$\sharp\{x \mid f(x) + tr(ux) = 0\} = 2^{2t-1} - \sharp\{x \mid f(x) + tr(ux) = 1\}$
Hence (E) becomes:

$\sharp\{x \mid f(x) + tr(ux) = 1\} = 2^{2t-1} - \sharp\{x \mid f(x) + tr(ux) = 1\}$
In other words $w(f + t_u) = 2^{2t-1} - w(f + t_u)$ and thus:
Conclusion:
If $tr(u\omega) + \epsilon = 1$ then $w(f + t_u) = 2^{2t-2}$ which is equivalent to $\hat{f}(u) = 0$.
For every ϵ the number of u such that $tr(u\omega) + \epsilon = 1$ is 2^{2t-2}. This is also the number of u such that $\hat{f}(u) = 0$ (see Proposition 1). Then, immediately: $\hat{f}(u) = 0$ if and only if $tr(u\omega) + \epsilon = 1$.

This means $\hat{I}_f = t_\omega + \epsilon$. □

2. RESULTS

The goal is to find all the r such that $[Q_u, Q_u + t_r]$ is bent or such that $[Q_u + Q_v, Q_u + Q_v + t_r]$ is bent.
Strategy:

For $f = Q_u$ or $f = Q_u + Q_u$, in order to apply Theorem 5(McGuire and Leander) we have to find $\hat{I}_f$ and $D_r(\hat{I}_f)$.

2.1. **The case.** $[Q_u, Q_u + t_r]$.

Theorem 8.

If $f = Q_u$ then $\hat{I}_f = \epsilon + t_{u^{-1}}$ with $\epsilon \in \mathbb{F}_2$.

Proof.
$Q_u(x) = \sum_{j=1}^{t-1} tr\big((ux)^{2^j+1}\big)$.
If $f_j(x) = (ux)^{2^j+1}$ then $D_{u^{-1}} f_j(x) = (ux)^{2^j+1} + [u(x+u^{-1})]^{2^j+1}$
$= ux + u^{2^j} x^{2^j} + 1$.
$tr(f_j(x)) = tr(x) + tr(u^{2^j} x^{2^j}) + tr(1) = tr(1) = 1$
$D_{u^{-1}} Q_u(x) = \sum_{j=1}^{t-1} D_{u^{-1}} f_j(x) = \sum_{j=1}^{t-1} 1 = t - 1 = \epsilon \in \mathbb{F}_2$.

According to the previous theorem: $\hat{I}_{Q_u} = t_{u^{-1}} + \epsilon$. □

Theorem 9. *Let u and r be in $\mathbb{F}_{2^{2t-1}}$.*

$[Q_u, Q_u + t_r]$ is bent if and only if $tr(u^{-1} r) = 1$.

Proof.
$D_r(\hat{I}_{Q_u})(x) = tr(u^{-1}x) + \epsilon + tr(u^{-1}(x + r)) + \epsilon$
$= tr(u^{-1}x) + tr(u^{-1}x) + tr(u^{-1}r)$
$= tr(u^{-1}r)$.

Then, from McGuire and Leander:
$[Q_u, Q_u + t_r]$ is bent if and only if $tr(u^{-1}r) = 1$. □

2.2. **The case.** $[Q_u + Q_v, Q_u + Q_v + t_r]$
Under the assumption on u and v then $[Q_u + Q_v, Q_u + Q_v + t_u + t_v]$ is a bent function. See Theorem 4, 5)
Hence $f(x) = Q_u + Q_v$ is near-bent (proposition 3).

Now we search $\omega \in F_{2^{2t-1}}$ such that $D_\omega f = \epsilon$ with $\epsilon \in \mathbb{F}_2$.

$D_\omega f = D_\omega Q_u + D_\omega Q_v$.

$Q_u(x) = \sum_{j=1}^{t-1} tr[f_{u,j}(x)]$ with $f_{u,j}(x) = (ux)^{2^j+1}$.

Since D_ω, $\sum$, tr are additive functions then:

$D_\omega Q_u = \sum_{j=1}^{t-1} tr[D_\omega f_{u,j}]$.

$D_\omega f_{u,j}(x) = u^{2^j+1} x^{2^j+1} + u^{2^j+1}(x + \omega)^{2^j+1}$.

$(x+\omega)^{2^j+1} = (x+\omega)^{2^j}(x+\omega) = (x^{2^j} + \omega^{2^j})(x + \omega)$.

$= x^{2^j+1} + \omega^{2^j} x + \omega x^{2^j} + \omega^{2^j+1}$.

$D_\omega f_{u,j}(x) = u^{2^j+1}(\omega^{2^j} x + \omega x^{2^j} + \omega^{2^j+1})$.

$D_\omega Q_u(x) = \sum_{j=1}^{t-1} tr[u^{2^j+1}(\omega^{2^j} x + \omega x^{2^j} + \omega^{2^j+1})]$.
$= \sum_{j=1}^{t-1} tr[u(u\omega)^{2^j} x] + \sum_{j=1}^{t-1} tr[u^{2^j+1}\omega x^{2^j})] + \sum_{j=1}^{t-1} tr[u^{2^j+1}\omega^{2^j+1})]$.
With $m = 2t - 1$ and since $x^m = x$ and $u^m = u$:

$\hat{f}(u) = \sum_{x \in \mathbb{F}_{2^{2t-1}}} (-1)^{f(x)+tr(ux)} = 2^{2t-1} - 2w(f + tr(ux)$. $\sum_{j=1}^{t-1} tr[u^{2^j+1}\omega x^{2^j})] = \sum_{j=1}^{t-1} tr[(u^{2^j+1}\omega x^{2^j})^{2^{m-j}}] = \sum_{j=1}^{t-1} tr[u(u\omega)^{2^{m-j}} x]$.

and thus:
$D_\omega Q_u(x) = \sum_{j=1}^{t-1} tr[u(u\omega)^{2^j}x] + \sum_{j=1}^{t-1} tr[u(u\omega)^{2^{m-j}}x] + \sum_{j=1}^{t-1} tr[u^{2^j+1}\omega^{2^j+1})]$.

When j runs from 1 to $t-1$ then $m-j$ runs from $2t-2$ to t.
Hence: $D_\omega Q_u = \sum_{j=1}^{t-1} tr[D_\omega f_{u,j}]$.

$D_\omega Q_u(x) = tr[u\sum_{j=1}^{2t-2}(u\omega)^{2^j}]x + \sum_{j=1}^{t-1} tr[u^{2^j+1}\omega^{2^j+1})]$.

By replacing u by v we find a similar result and finally:

$D_\omega f(x) = tr\big([u\sum_{j=1}^{2t-2}(u\omega)^{2^j} + v\sum_{j=1}^{2t-2}(v\omega)^{2^j}]x\big) + \epsilon$ with $\epsilon \in \mathbb{F}_2$.

$\hat{f}(u) = \sum_{x\in\mathbb{F}_{2^{2t-1}}}(-1)^{f(x)+tr(ux)} = 2^{2t-1} - 2w(f+tr(ux)$. It follows that $D_\omega f$ is a constant function if and only if

$$(*) \quad u\sum_{j=1}^{2t-2}(u\omega)^{2^j} + v\sum_{j=1}^{2t-2}(v\omega)^{2^j} = 0.$$

Remark that $\sum_{j=1}^{2t-2}(u\omega)^{2^j} = u\omega + tr(u\omega)$ and $\sum_{j=1}^{2t-2}(v\omega)^{2^j} = v\omega + tr(v\omega)$. Then $(*)$ becomes:

$$(*) \quad (u^2+v^2)\omega + utr(u\omega) + vtr(v\omega) = 0.$$

Case 1: $tr(u\omega) = tr(v\omega) = 0$.
we find the trivial solution $\omega = 0$.
Case 2: $tr(u\omega) = tr(v\omega) = 1$.
$\omega = (u+v)^{-1}$ and $tr(u\omega) = tr[u(u+v)^{-1}]$, $tr(v\omega) = tr[v(u+v)^{-1}]$.
This leads to $tr(u\omega) + tr(v\omega) = tr[(u+v)(u+v)^{-1}] = tr(1) = 1$if $tr(u^{-1}r) = 1$.
which is imposible because $tr(u\omega) = tr(v\omega)$.

Case 3: $tr(u\omega) = 1$, $tr(v\omega) = 0$,
$D_\omega Q_u = \sum_{j=1}^{t-1} tr[D_\omega f_{u,j}]$. $\omega = u(u^2+v^2)^{-1}$

Case 4: $tr(u\omega) = 0$, $tr(v\omega) = 1$.
$\omega = v(u^2+v^2)^{-1}$.

In case 3, $tr(u\omega) = tr(u^2(u^2+v^2)^{-1}) = tr[(u(u+v)^{-1})^2]$.
$= tr[u(u+v)^{-1}]$. Similarly in case 4:
$tr(v\omega) = tr[v(u+v)^{-1}]$. Then $tr[u(u+v)^{-1}] = tr[v(u+v)^{-1}]$ is impossible since
$tr(u^{-1}r) = 1$. $tr[u(u+v)^{-1}] + tr[v(u+v)^{-1}] = tr((u+v)(u+v)^{-1}) = tr(1) = 1$. Conclusion:

Proposition 10. $f = Q_u + Q_v$.
If ω is a non-zero element such that $D_\omega f = \epsilon$ with $\epsilon \in \mathbb{F}_2$ then:

$\omega = u(u^2+v^2)^{-1}$ *if* $tr[u(u+v)^{-1}] = 1$.

$\omega = v(u^2+v^2)^{-1}$ *if* $tr[v(u+v)^{-1}] = 1$.

We are now in position to find all $e \in \mathbb{F}_{2^{2t-1}}$ such that $[f, f+t_e]$ is a bent function and consider the case $e = r+s$

Theorem 11.
Let $u \neq 0$, $tr(u^{-1}r) = 1$, $v \neq 0$, $tr(v^{-1}s) = 1, u \neq v$, $r \neq s$.
Define ω *by:*
$\omega = u(u^2 + v^2)^{-1}$ *if* $tr(u(u + v)^{-1}) = 1$.
$\omega = v(u^2 + v^2)^{-1}$ *if* $tr(v(u + v)^{-1}) = 1$.

If $tr(\omega(r + s)) = 1$ *then:*

$$[Q_u, t_r] + [Q_v, t_s]$$

is a bent function.

Proof.
Applying Theorem 7, since $D_\omega f = \epsilon$ then $\hat{I}_f = t_\omega + \epsilon$. Now, if $e \in \mathbb{F}_{2^{2t-1}}$ then $D_e\hat{I}_f(x) = \hat{I}_f(x) + \hat{I}_f(x + e) = tr(\omega x + tr(\omega(x + e) = tr(\omega x) + tr(\omega x) + tr(\omega e) = tr(\omega e)$. Hence $D_e\hat{I}_f(x) = 1$ if and only if $tr(\omega e) = 1$. From Theorem 5, $[f, f + t_e]$ is a bent function if and only if $tr(\omega e) = 1$. Now if $f = Q_u + Q_v$ then $[f, f + t_{r+s}] = [Q_u, Q_u + r] + [Q_v, Q_v + s]$ is a bent function if and only if $tr(\omega(r + s)) = 1$. □

3. Another construction.

Theorem 12.
Let γ *be in* $\mathbb{F}_{2^{2t-1}}, tr(u^{-1}r) = 1$ *then:*
$[Q_u + t_1t_\gamma, Q_u + t_r + t_1t_\gamma]$ *is a bent function.*

Proof.
This is a special case of Theorem 20 of [10] with $f_0 = Q_u$ and
$f_1 = Q_u + t_r$ □

Examples:
$[Q_u + t_1t_\gamma, Q_u + t_r + t_1t_\gamma]$ with $tr(u^{-1}r) = 1$.

$[Q_u + Q_v + t_1t_\gamma, Q_u + Q_v + t_{r+s} + t_1t_\gamma]$ with conditions of Theorem 11 on u, v, r, s.

4. Conclusions

By using a slight modification of Kerdock bent functions we have introduced new bent functions.

The number of new bent functions $[Q_u, Q_u + t_r]$ (Theorem 9) is greater than the number of Kerdock bent functions $[Q_u, Q_u + t_u]$,.
The number of new bent functions $[Q_u + t_1t_\gamma, Q_u + t_r + t_1t_\gamma]$,
$u \neq 0, tr(u^{-1}r) = 1, \gamma \neq 0$ (Theorem 12) is greater than the number of Kerdock bent functions $[Q_u, Q_u + t_r]$.
It is easy to check that:

Bent Functions	Number
$[Q_u, Q_u + t_u], u \neq 0$	$(2^{2t-1} - 1)$
$[Q_u, Q_u + t_r]$ with $tr(u^{-1}r) = 1$	$2^{2t-2}(2^{2t-1} - 1)$
$[Q_u + t_1 t_\gamma, Q_u + t_r + t_1 t_\gamma]$	$2^{4t-3}(2^{2t-1} - 1)$
$[Q_u, Q_u + t_u]$ $+[Q_v, Q_v + t_v]$	$(2^{2t-1} - 1)^2$
$[Q_u, Q_u + t_r]$ $+[Q_v, Q_v + t_s]$	$A(2^{2t-1} - 1)^2$

with $A = \sharp\{(r, s) \mid tr(u^{-1}r) = 1, tr(u^{-1}s) = 1, tr(\omega(r + s) = 1\}$.
(Notations of Theorem 11.)

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