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KERDOCK-LIKE BENT FUNCTIONS

J.WOLFMANN

ABSTRACT. We introduce bent functions similar to bent functions whose binary representative vectors are members of the famous Kerdock code.

1. INTRODUCTION

1.1. Elementary definitions.

\mathbb{F}_2 is the finite field of order 2.

A m -boolean function is a map from \mathbb{F}_2^m into \mathbb{F}_2 .

Weight: $w(F) = \#\{v \in \mathbb{F}_2^m \mid F(v) = 1\}$.

Derivative: $e \in \mathbb{F}_2^m$ $(D_e F)(X) = F(X) + F(X + e)$.

Fourier coefficients:

$\hat{F}(v) = \sum_{X \in \mathbb{F}_2^m} (-1)^{F(X) + \langle v, X \rangle}$ where \langle, \rangle inner product of \mathbb{F}_2^m .

The set $\{\hat{F}(v) \mid v \in \mathbb{F}_{2^m}\}$ is independant of the choice of \langle, \rangle .

Definitions:

F is bent if: $\forall v \in \mathbb{F}_2^m : \hat{F}(v)$ is in $\{-2^{m/2}, 2^{m/2}\}$.

Exist only when m is even.

F is near-benf if: $\forall v \in \mathbb{F}_{2^m} : \hat{F}(v)$ is in $\{-2^{(m+1)/2}, 0, 2^{(m+1)/2}\}$.

Exist only when m is odd.

Bent functions were introduced by Rothaus in [6]. They are interesting for Coding Theory, Cryptology and Sequences and were the topic of a lot of works. See for instance [2], [5] Chap. 14, [7], [1].

For further use we need the following Proposition.

Proposition 1. *The distribution of the Fourier coefficients of a $(2t - 1)$ -near bent function f is:*

$$\begin{aligned} \hat{f}(v) = 2^t & \quad \text{number of } v: 2^{2t-3} + (-1)^{f(0)} 2^{t-2} \\ \hat{f}(v) = 0 & \quad \text{number of } v: 2^{2t-2} \\ \hat{f}(v) = -2^t & \quad \text{number of } v: 2^{2t-3} - (-1)^{f(0)} 2^{t-2}. \end{aligned}$$

Proof. See Proposition 4 in [1]). □

1.2. Special representations of boolean functions.

1) Using finite fields:

\mathbb{F}_2^m identified with the field \mathbb{F}_{2^m} .

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In this case the inner product \langle, \rangle of \mathbb{F}_{2^m} is defined by:
 $\langle a, x \rangle = \text{tr}(ax)$ where tr is the trace of \mathbb{F}_{2^m} over \mathbb{F}_2 .

2) Representative vector (truth table)

Indexing \mathbb{F}_{2^m} with any order $e_0, e_1, \dots, e_{2^m-1}$ the representative vector of a m -boolean function F is the binary vector $(F(e_i))_{i=0}^{2^m-1}$.

This vector depends on the choice of the order of \mathbb{F}_{2^m} .

3) A two-variable representation.

This is the representation chosen by Kerdock to introduce his famous code.

We identify $\mathbb{F}_{2^{2t}}$ with the product:

$$\mathbb{F}_{2^{2t-1}} \times \mathbb{F}_2 = \{X = (u, \nu) \mid u \in \mathbb{F}_{2^{2t-1}}, \nu \in \mathbb{F}_2\}.$$

If F is a $(2t)$ -boolean function then define two $(2t-1)$ -boolean functions f_0, f_1 , such that $f_0(u) = F(u, 0)$ and $f_1(u) = F(u, 1)$.

The two-variable representation (TVR) of F is defined by the 2-variable polynomial: $\phi_F(x, y) = (y + 1)f_0(x) + yf_1(x)$

This is a representation of F in the following sense. Since:

$$\phi_F(u, 0) = f_0(u) = F(u, 0), \quad \phi_F(u, 1) = f_1(u) = F(u, 1).$$

then if $X = (u, \nu)$, with $u = 0$ or $u = 1$: $F(X) = \phi_F(u, \nu)$.

$$\text{Notation: } F = [f_0, f_1]$$

Let α be a primitive root of $\mathbb{F}_{2^{k-1}}$. As order of $\mathbb{F}_{2^{2t}}$ we choose:

$$\mathbb{F}_{2^{2t}} : (0, 0), (\alpha^0, 0) \dots (\alpha^i, 0) \dots (\alpha^{n/2-2}, 0) \mid (0, 1), (\alpha^0, 1) \dots (\alpha^i, 1) \dots (\alpha^{n/2-2}, 1)$$

The representative vector of $F = [f_0, f_1]$ is:

$$(f_0(0) \dots f_0(\alpha^i) \dots f_0(\alpha^{n/2-2}) \quad f_1(0) \dots f_1(\alpha^i) \dots f_1(\alpha^{n/2-2}))$$

1.3. From Near-bent to Bent.

We now characterize the f_0, f_1 when $F = [f_0, f_1]$ is bent.

Proposition 2. (well known)

A $(2t)$ -boolean function $F = [f_0, f_1]$ is a bent if and only if:

(a) f_0 and f_1 are near-bent.

(b) $\forall u \in \mathbb{F}_{2^{2t-1}} \mid \hat{f}_0(u) \mid + \mid \hat{f}_1(u) \mid = 2^t$

Proof. A proof is given in [9], Proposition 14. □

Remark: (b) means that one of $\mid \hat{f}_0(a) \mid$ and $\mid \hat{f}_1(a) \mid$ is equal to 2^t and the other one is equal to 0.

1.4. The Kerdock code. :

Notation:

$$Q(x) = \sum_{j=1}^{t-1} \text{tr}(x^{2^j+1}).$$

If $e \in \mathbb{F}_{2^m}$: $t_e(x) = \text{tr}(ex)$, $Q_e(x) = Q(ex)$.

Definition 3.

The Kerdock code of length 2^{2t} is the set of the representative vectors of the $2t$ -boolean functions

$$F = [Q_u, Q_u + t_u] + \text{affine-linear form}$$

with $u \in \mathbb{F}_{2^{2t-1}}$.

Example: The representative vector of $F = [Q_u, Q_u + t_u]$ is:

$$m = (f_0(0) \dots \dots f_0(\alpha^i) \dots \dots f_0(\alpha^{n/2-2}) \quad f_1(0) \dots \dots f_1(\alpha^i) \dots \dots f_1(\alpha^{n/2-2}))$$

with $f_0(x) = Q_u(x)$ and $f_1(x) = (Q_u + t_u)(x)$

Theorem 4. (Kerdock)

With the above notations:

If $u \neq 0$ then:

$F = [Q_u, Q_u + t_u] + \text{affine-linear form}$ is a Bent Function.

Proof. See [3] or [5] chapter 15. □

Remark:

From the elementary properties of bent functions, $F = [Q_u, Q_u + t_u] + \text{affine-linear form}$ is bent if and only if $[Q_u, Q_u + t_u]$ is bent. Hence we restrict our research to $[Q_u, Q_u + t_u]$.

Definition: For the sequel of the paper $[Q_u, Q_u + t_u]$ is called a Kerdock bent function.

The Kerdock code K_{2t} is a binary non-linear code with several interesting properties. For instance:

- 1) $[Q_u, Q_u + t_u]$ is a bent functions
- 2) $[Q_u, Q_u + t_u] + [Q_v, Q_v + t_v]$ is a bent function.

A problem:

The question of this paper is to replace t_u in $[Q_u, Q_u + t_u]$ by another $2t - 1$ -boolean function, for example t_r , to get another bent function.

1.5. Main tools.

Definition 5.

If f is a $(2t - 1)$ -near-bent function then \hat{I}_f is the indicator of the set $\{x \in \mathbb{F}_{2^{2t-1}} \mid \hat{f}(x) = 0\}$ where \hat{f} is the Fourier transform of f .

(In other words, $\hat{I}_f(x) = 1$ if and only if $\hat{f}(x) = 0$).

The two Theorems below are the main tools of the present work.

Theorem 6. (*McGuire and Leander*)

Let f be a $(2t - 1)$ -near-bent function and let v be in $\mathbb{F}_{2^{2t-1}}$.
 $D_v(\hat{I}_f) = 1$ if and only if $[f, f + t_v]$ is a bent-function.

Proof. See [4], Theorem 3. □

Theorem 7. (*W*)

Let f be a $(2t - 1)$ -near-bent function.

Let ω be in $\mathbb{F}_{2^{2t-1}}$ and let ϵ be in \mathbb{F}_2 .

If $D_\omega f = \epsilon$ then $\hat{I}_f = t_\omega + \epsilon$.

Remark: According to the definition of \hat{I}_f this lemma means that if $D_\omega f = \epsilon$ then $\hat{f}(x) = 0$ if and only if $t_\omega(x) = 1 + \epsilon$.

Proof. $\hat{f}(u) = \sum_{x \in \mathbb{F}_{2^{2t-1}}} (-1)^{f(x) + tr(ux)} = 2^{2t-1} - 2w(f + tr(ux))$.

$\hat{f}(u) = 0$ if and only if $w(f + t_u) = 2^{2t-2}$.

$D_\omega f = \epsilon$ means that $f(x + \omega) = f(x) + \epsilon$.

The transform $\tau : x \rightarrow x + \omega$ is a permutation of $\mathbb{F}_{2^{2t-1}}$ and then preserves the weight of every $(2t - 1)$ -Boolean function. Thus:

$$\#\{x \mid f(x) + tr(ux) = 1\} = \#\{x \mid f(x + \omega) + tr(u(x + \omega)) = 1\}.$$

$$(E) \#\{x \mid f(x) + tr(ux) = 1\} = \#\{x \mid f(x) + \epsilon + tr(ux) + tr(u\omega) = 1\}.$$

Now assume $tr(u\omega) + \epsilon = 1$. The right hand member of (E) is:

$$\#\{x \mid f(x) + tr(ux) = 0\} = 2^{2t-1} - \#\{x \mid f(x) + tr(ux) = 1\}$$

Hence (E) becomes:

$$\#\{x \mid f(x) + tr(ux) = 1\} = 2^{2t-1} - \#\{x \mid f(x) + tr(ux) = 1\}$$

In other words $w(f + t_u) = 2^{2t-1} - w(f + t_u)$ and thus:

Conclusion:

If $tr(u\omega) + \epsilon = 1$ then $w(f + t_u) = 2^{2t-2}$ which is equivalent to $\hat{f}(u) = 0$.

For every ϵ the number of u such that $tr(u\omega) + \epsilon = 1$ is 2^{2t-2} . This is also the number of u such that $\hat{f}(u) = 0$ (see Proposition 1). Then, immediately: $\hat{f}(u) = 0$ if and only if $tr(u\omega) + \epsilon = 1$.

This means $\hat{I}_f = t_\omega + \epsilon$. □

2. RESULTS

The goal is to find all the r such that $[Q_u, Q_u + t_r]$ is bent or such that $[Q_u + Q_v, Q_u + Q_v + t_r]$ is bent.

Strategy:

For $f = Q_u$ or $f = Q_u + Q_v$, in order to apply Theorem 5(McGuire and Leander) we have to find \hat{I}_f and $D_r(\hat{I}_f)$.

2.1. **The case.** $[Q_u, Q_u + t_r]$.

Theorem 8.

If $f = Q_u$ then $\hat{I}_f = \epsilon + t_{u^{-1}}$ with $\epsilon \in \mathbb{F}_2$.

Proof.

$$Q_u(x) = \sum_{j=1}^{t-1} \text{tr}((ux)^{2^j+1}).$$

$$\begin{aligned} \text{If } f_j(x) &= (ux)^{2^j+1} \text{ then } D_{u^{-1}}f_j(x) = (ux)^{2^j+1} + [u(x + u^{-1})]^{2^j+1} \\ &= ux + u^{2^j}x^{2^j} + 1. \end{aligned}$$

$$\text{tr}(f_j(x)) = \text{tr}(x) + \text{tr}(u^{2^j}x^{2^j}) + \text{tr}(1) = \text{tr}(1) = 1$$

$$D_{u^{-1}}Q_u(x) = \sum_{j=1}^{t-1} D_{u^{-1}}f_j(x) = \sum_{j=1}^{t-1} 1 = t - 1 = \epsilon \in \mathbb{F}_2.$$

According to the previous theorem: $\hat{I}_{Q_u} = t_{u^{-1}} + \epsilon$. □

Theorem 9. Let u and r be in $\mathbb{F}_{2^{2t-1}}$.

$[Q_u, Q_u + t_r]$ is bent if and only if $\text{tr}(u^{-1}r) = 1$.

Proof.

$$\begin{aligned} D_r(\hat{I}_{Q_u})(x) &= \text{tr}(u^{-1}x) + \epsilon + \text{tr}(u^{-1}(x + r)) + \epsilon \\ &= \text{tr}(u^{-1}x) + \text{tr}(u^{-1}x) + \text{tr}(u^{-1}r) \\ &= \text{tr}(u^{-1}r). \end{aligned}$$

Then, from McGuire and Leander:

$[Q_u, Q_u + t_r]$ is bent if and only if $\text{tr}(u^{-1}r) = 1$. □

2.2. **The case.** $[Q_u + Q_v, Q_u + Q_v + t_r]$

Under the assumption on u and v then $[Q_u + Q_v, Q_u + Q_v + t_u + t_v]$ is a bent function. See Theorem 4, 5)

Hence $f(x) = Q_u + Q_v$ is near-bent (proposition 3).

Now we search $\omega \in F_{2^{2t-1}}$ such that $D_\omega f = \epsilon$ with $\epsilon \in \mathbb{F}_2$.

$$D_\omega f = D_\omega Q_u + D_\omega Q_v.$$

$$Q_u(x) = \sum_{j=1}^{t-1} \text{tr}[f_{u,j}(x)] \text{ with } f_{u,j}(x) = (ux)^{2^j+1}.$$

Since $D_\omega, \sum, \text{tr}$ are additive functions then:

$$D_\omega Q_u = \sum_{j=1}^{t-1} \text{tr}[D_\omega f_{u,j}].$$

$$D_\omega f_{u,j}(x) = u^{2^j+1}x^{2^j+1} + u^{2^j+1}(x + \omega)^{2^j+1}.$$

$$(x + \omega)^{2^j+1} = (x + \omega)^{2^j}(x + \omega) = (x^{2^j} + \omega^{2^j})(x + \omega).$$

$$= x^{2^j+1} + \omega^{2^j}x + \omega x^{2^j} + \omega^{2^j+1}.$$

$$D_\omega f_{u,j}(x) = u^{2^j+1}(\omega^{2^j}x + \omega x^{2^j} + \omega^{2^j+1}).$$

$$D_\omega Q_u(x) = \sum_{j=1}^{t-1} \text{tr}[u^{2^j+1}(\omega^{2^j}x + \omega x^{2^j} + \omega^{2^j+1})].$$

$$= \sum_{j=1}^{t-1} \text{tr}[u(u\omega)^{2^j}x] + \sum_{j=1}^{t-1} \text{tr}[u^{2^j+1}\omega x^{2^j}] + \sum_{j=1}^{t-1} \text{tr}[u^{2^j+1}\omega^{2^j+1}].$$

With $m = 2t - 1$ and since $x^m = x$ and $u^m = u$:

$$\begin{aligned} \hat{f}(u) &= \sum_{x \in \mathbb{F}_{2^{2t-1}}} (-1)^{f(x) + \text{tr}(ux)} = 2^{2t-1} - 2w(f + \text{tr}(ux)) \cdot \sum_{j=1}^{t-1} \text{tr}[u^{2^j+1}\omega x^{2^j}] = \\ &= \sum_{j=1}^{t-1} \text{tr}[(u^{2^j+1}\omega x^{2^j})^{2^{m-j}}] = \sum_{j=1}^{t-1} \text{tr}[u(u\omega)^{2^{m-j}}x]. \end{aligned}$$

and thus:

$$D_\omega Q_u(x) = \sum_{j=1}^{t-1} \text{tr}[u(u\omega)^{2^j} x] + \sum_{j=1}^{t-1} \text{tr}[u(u\omega)^{2^{m-j}} x] + \sum_{j=1}^{t-1} \text{tr}[u^{2^j+1} \omega^{2^j+1}].$$

When j runs from 1 to $t-1$ then $m-j$ runs from $2t-2$ to t .

$$\text{Hence: } D_\omega Q_u = \sum_{j=1}^{t-1} \text{tr}[D_\omega f_{u,j}].$$

$$D_\omega Q_u(x) = \text{tr}[u \sum_{j=1}^{2t-2} (u\omega)^{2^j} x] + \sum_{j=1}^{t-1} \text{tr}[u^{2^j+1} \omega^{2^j+1}].$$

By replacing u by v we find a similar result and finally:

$$D_\omega f(x) = \text{tr}([u \sum_{j=1}^{2t-2} (u\omega)^{2^j} + v \sum_{j=1}^{2t-2} (v\omega)^{2^j}]x) + \epsilon \text{ with } \epsilon \in \mathbb{F}_2.$$

$\hat{f}(u) = \sum_{x \in \mathbb{F}_{2^{2t-1}}} (-1)^{f(x)+\text{tr}(ux)} = 2^{2t-1} - 2w(f + \text{tr}(ux))$. It follows that $D_\omega f$ is a constant function if and only if

$$(*) \quad u \sum_{j=1}^{2t-2} (u\omega)^{2^j} + v \sum_{j=1}^{2t-2} (v\omega)^{2^j} = 0.$$

Remark that $\sum_{j=1}^{2t-2} (u\omega)^{2^j} = u\omega + \text{tr}(u\omega)$ and $\sum_{j=1}^{2t-2} (v\omega)^{2^j} = v\omega + \text{tr}(v\omega)$. Then $(*)$ becomes:

$$(*) \quad (u^2 + v^2)\omega + u\text{tr}(u\omega) + v\text{tr}(v\omega) = 0.$$

Case 1: $\text{tr}(u\omega) = \text{tr}(v\omega) = 0$.

we find the trivial solution $\omega = 0$.

Case 2: $\text{tr}(u\omega) = \text{tr}(v\omega) = 1$.

$\omega = (u+v)^{-1}$ and $\text{tr}(u\omega) = \text{tr}[u(u+v)^{-1}]$, $\text{tr}(v\omega) = \text{tr}[v(u+v)^{-1}]$. This leads to $\text{tr}(u\omega) + \text{tr}(v\omega) = \text{tr}[(u+v)(u+v)^{-1}] = \text{tr}(1) = 1$ if $\text{tr}(u^{-1}v) = 1$.

which is impossible because $\text{tr}(u\omega) = \text{tr}(v\omega)$.

Case 3: $\text{tr}(u\omega) = 1$, $\text{tr}(v\omega) = 0$,

$$D_\omega Q_u = \sum_{j=1}^{t-1} \text{tr}[D_\omega f_{u,j}]. \quad \omega = u(u^2 + v^2)^{-1}$$

Case 4: $\text{tr}(u\omega) = 0$, $\text{tr}(v\omega) = 1$.

$$\omega = v(u^2 + v^2)^{-1}.$$

In case 3, $\text{tr}(u\omega) = \text{tr}(u^2(u^2 + v^2)^{-1}) = \text{tr}[(u(u+v)^{-1})^2]$.

$= \text{tr}[u(u+v)^{-1}]$. Similarly in case 4:

$\text{tr}(v\omega) = \text{tr}[v(u+v)^{-1}]$. Then $\text{tr}[u(u+v)^{-1}] = \text{tr}[v(u+v)^{-1}]$ is impossible since

$\text{tr}(u^{-1}v) = 1$. $\text{tr}[u(u+v)^{-1}] + \text{tr}[v(u+v)^{-1}] = \text{tr}((u+v)(u+v)^{-1}) = \text{tr}(1) = 1$. Conclusion:

Proposition 10. $f = Q_u + Q_v$.

If ω is a non-zero element such that $D_\omega f = \epsilon$ with $\epsilon \in \mathbb{F}_2$ then:

$$\omega = u(u^2 + v^2)^{-1} \text{ if } \text{tr}[u(u+v)^{-1}] = 1.$$

$$\omega = v(u^2 + v^2)^{-1} \text{ if } \text{tr}[v(u+v)^{-1}] = 1.$$

We are now in position to find all $e \in \mathbb{F}_{2^{2t-1}}$ such that $[f, f + t_e]$ is a bent function and consider the case $e = r + s$

Theorem 11.

Let $u \neq 0$, $\text{tr}(u^{-1}r) = 1$, $v \neq 0$, $\text{tr}(v^{-1}s) = 1$, $u \neq v$, $r \neq s$.

Define ω by:

$$\omega = u(u^2 + v^2)^{-1} \text{ if } \text{tr}(u(u + v)^{-1}) = 1.$$

$$\omega = v(u^2 + v^2)^{-1} \text{ if } \text{tr}(v(u + v)^{-1}) = 1.$$

If $\text{tr}(\omega(r + s)) = 1$ then:

$$[Q_u, t_r] + [Q_v, t_s]$$

is a bent function.

Proof.

Applying Theorem 7, since $D_\omega f = \epsilon$ then $\hat{I}_f = t_\omega + \epsilon$. Now, if $e \in \mathbb{F}_{2^{2t-1}}$ then $D_e \hat{I}_f(x) = \hat{I}_f(x) + \hat{I}_f(x + e) = \text{tr}(\omega x + \text{tr}(\omega(x + e)) = \text{tr}(\omega x) + \text{tr}(\omega x) + \text{tr}(\omega e) = \text{tr}(\omega e)$. Hence $D_e \hat{I}_f(x) = 1$ if and only if $\text{tr}(\omega e) = 1$. From Theorem 5, $[f, f + t_e]$ is a bent function if and only if $\text{tr}(\omega e) = 1$. Now if $f = Q_u + Q_v$ then $[f, f + t_{r+s}] = [Q_u, Q_u + r] + [Q_v, Q_v + s]$ is a bent function if and only if $\text{tr}(\omega(r + s)) = 1$. \square

3. ANOTHER CONSTRUCTION.

Theorem 12.

Let γ be in $\mathbb{F}_{2^{2t-1}}$, $\text{tr}(u^{-1}r) = 1$ then:

$[Q_u + t_1 t_\gamma, Q_u + t_r + t_1 t_\gamma]$ is a bent function.

Proof.

This is a special case of Theorem 20 of [10] with $f_0 = Q_u$ and $f_1 = Q_u + t_r$ \square

Examples:

$[Q_u + t_1 t_\gamma, Q_u + t_r + t_1 t_\gamma]$ with $\text{tr}(u^{-1}r) = 1$.

$[Q_u + Q_v + t_1 t_\gamma, Q_u + Q_v + t_{r+s} + t_1 t_\gamma]$ with conditions of Theorem 11 on u, v, r, s .

4. CONCLUSIONS

By using a slight modification of Kerdock bent functions we have introduced new bent functions.

The number of new bent functions $[Q_u, Q_u + t_r]$ (Theorem 9) is greater than the number of Kerdock bent functions $[Q_u, Q_u + t_u]$.

The number of new bent functions $[Q_u + t_1 t_\gamma, Q_u + t_r + t_1 t_\gamma]$, $u \neq 0$, $\text{tr}(u^{-1}r) = 1$, $\gamma \neq 0$ (Theorem 12) is greater than the number of Kerdock bent functions $[Q_u, Q_u + t_r]$.

It is easy to check that:

Bent Functions	Number
$[Q_u, Q_u + t_u], u \neq 0$	$(2^{2t-1} - 1)$
$[Q_u, Q_u + t_r]$ with $tr(u^{-1}r) = 1$	$2^{2t-2}(2^{2t-1} - 1)$
$[Q_u + t_1 t_\gamma, Q_u + t_r + t_1 t_\gamma]$	$2^{4t-3}(2^{2t-1} - 1)$
$[Q_u, Q_u + t_u] + [Q_v, Q_v + t_v]$	$(2^{2t-1} - 1)^2$
$[Q_u, Q_u + t_r] + [Q_v, Q_v + t_s]$	$A(2^{2t-1} - 1)^2$
with $A = \#\{(r, s) \mid tr(u^{-1}r) = 1, tr(u^{-1}s) = 1, tr(\omega(r+s)) = 1\}$.	
(Notations of Theorem 11.)	

5. REFERENCES

- [1] A.Canteault, P.Charpin, Decomposing Bent Functions
IEEE Transactions on Information Theory, vol.49, **8**, (2003),
2004-2019.
- [2] J.F.Dillon, Elementary Hadamard Difference Sets.
bent (Kerdock).
Ph.D. Thesis, University of Maryland(1974).
- [3] A. M.Kerdock A class of low-rate non linear codes.
Information and Control 20, pp. 182-187, 1972.
- [4] G. Leander, G. McGuire, Construction of Bent Functions from Near-Bent Functions.
Journal of Combinatorial Theory, Series A, vol.116, **4**, (2009), 960-970.
- [5] F.J.Mac Williams, N.J.A.Sloane
The Theory of Error Correcting Codes, North-Holland, Amsterdam,
1977.
- [6] O.S.Rothaus, On Bent Functions.
Journal of Combinatorial Theory, series A, 20, (1976), 300-305.
- [7] J.Wolfmann, Bent Functions and Coding Theory.
in *Difference Sets, Sequences and their Correlation properties*
(A. Pott, P.V. Kumar, T. Helleseeth, D. Jungnickel, Eds),
NATO Sciences Series.
Series C, vol.542, Kluwer Academic Publishers (1999) 393-418.
- [8] J. Wolfmann, Cyclic code aspects of bent functions, in *Finite Fields: Theory and Applications*, AMS series "Contemporary Mathematics"
volume 518, 363-384, 2010
- [9] J. Wolfmann, Special Bent and Near-Bent Functions
in *Advances in Mathematics of Communication*, vol.8, No 1 (2014), 21-33

[10] J. Wolfmann, From Near-Bent to Bent: A special Case. In *Topics in Finite Fields*, AMS series "Contemporary Mathematics" volume 632, 359-371, 2015

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